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# Composition operators between Fréchet spaces of holomorphic functions

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**Abstract.** Let  $E$ ,  $F$  and  $G$  be Banach spaces. Let  $V$  a balanced open subset of  $F$ . The reflexive and Montel composition operator  $T_\Phi(f) := f \circ \Phi$  acting between the Fréchet spaces of all  $G$ -valued holomorphic functions of bounded type on  $E$  is studied in terms of  $\Phi$ , where  $\Phi$  is a  $G$ -valued holomorphic functions of bounded type on  $V$ .

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## 1 Introduction

Let  $E$  and  $G$  be complex Banach spaces. For an open subset  $U$  of  $E$ ,  $H_b(U, G)$  denotes the space of all holomorphic functions from  $U$  into  $G$  which are bounded on  $U$ -bounded subsets of  $U$ . It is endowed with the topology  $\tau_b$  of uniform convergence on  $U$ -bounded sets. It is known that  $H_b(U, G)$  is a Fréchet space. As usual, we will always omit  $G$  in the notation in case  $G = \mathbb{C}$ . So, for instance, we will write  $H_b(U)$  for  $H_b(U, \mathbb{C})$ .

If  $F$  is a complex Banach space and  $V$  an open subset of  $F$ , given a holomorphic mapping of bounded type  $\Phi : V \rightarrow E$  with  $\Phi(V) \subset U$  we will consider the composition operator  $T_\Phi : H_b(U, G) \rightarrow H_b(V, G)$  defined by  $T_\Phi(f) = f \circ \Phi$ . Recently the study of composition operators has deserved some attention. Several results of composition operators between the Fréchet spaces  $H_b(U, G)$  of holomorphic functions of bounded type have appeared when  $G = \mathbb{C}$  (see for example [2], [4]). For instance, in [3] and [4] M. González and J. Gutiérrez have found results relating the (weak) compactness of holomorphic function

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of bounded type  $\Phi \in H_b(F, E)$  with the (weak) compactness of composition operator  $T_\Phi$  from  $H_b(E)$  into  $H_b(V)$ .

It seems natural to study Montel and reflexive composition operators between the Fréchet spaces  $H_b(U, G)$  of all vector valued holomorphic functions of bounded type.

In this note we study reflexive and Montel composition operators  $T_\Phi$  between the Fréchet spaces of all  $G$ -valued holomorphic functions of bounded type in terms of  $\Phi$ .

Let us mention that if  $G$  is a complex Banach algebra the space  $H_b(U, G)$  endowed the  $\tau_b$  topology of the uniform convergence on  $U$ -bounded set is a Fréchet algebra and the composition operator  $T_\Phi$  is a continuous homomorphism.

**Preliminaries.** Our notation is standard and we refer to the books of Dineen [1] and Mujica [7] for background information on holomorphic functions on infinite dimensional Banach spaces, to Jarchow [6] and Horvath [5] regarding locally convex spaces theory.

Let  $E$  and  $G$  be complex Banach spaces. If  $U$  is an open subset of  $E$ , then a set  $A \subset U$  is said to be  $U$ -bounded if  $A$  is bounded and is bounded away from the boundary of  $U$ .

If  $X$  and  $Y$  are complex Hausdorff locally convex spaces and  $T : X \rightarrow Y$  is a linear map, then  $T^* : Y^* \rightarrow X^*$  defined by  $T^*(y^*) = y^* \circ T$  is a well defined linear map, it is called the algebraic adjoint of  $T$ . Under some conditions  $T^*$  induces a map  $T' : Y' \rightarrow X'$  and we call  $T'$  the adjoint or transposed map of  $T$ .

A continuous linear map from  $X$  into  $Y$  is called *Reflexive* (resp. *Montel*), if it transforms bounded sets into relatively weakly compact (resp. relatively compact) sets.

Let us recall that a continuous linear mappings  $T$  from  $X$  into  $Y$  is called *weakly compact* (resp. *compact*), if it maps some 0-neighborhood into relatively weakly compact (resp. relatively compact) sets. If  $X$  and  $Y$  are normed space  $T$  is weakly compact (resp. compact) if and only if  $T$  is Reflexive (resp. Montel).

Let now  $X$  be a complex Hausdorff locally convex space and let  $X'$  be its topological dual. As usual  $\sigma(X, X')$  is the weak topology on  $X$  and  $\sigma(X', X)$  is the weak-star topology on  $X'$ .  $\tau_\beta$  denotes the strong topology on  $X$ ,  $\tau_\mu$  denotes the Mackey topology on  $X'$  and  $\tau_c$  denotes the topology of the uniform convergence on compact subsets of  $X$  on  $X'$ .

We will denote  $\mathcal{B}_X(0)$  a fundamental system of neighborhoods of 0 of the Hausdorff locally convex space  $X$ .

Let  $\mathcal{A}$  a bounded set in  $H_b(U, G)$ , we associate the following neighborhood

of 0 in  $(H_b(U, G)', \tau_\beta)$ :

$$U_{0, \mathcal{A}, \epsilon} = \{ f' \in H_b(U, G)' / \sup_{f \in \mathcal{A}} |f'(f)| < \epsilon \} \in \mathcal{B}_{(H_b(U, G)', \tau_\beta)}(0).$$

For each finite subset  $\{g_1, \dots, g_r\}$  of  $H_b(U, G)$  and for each  $\epsilon > 0$  we consider the following neighborhood of 0

$$V_{0, g_1, \dots, g_r, \epsilon} = \{ f' \in H_b(U, G)' / |f'(g_1)| < \epsilon, \dots, |f'(g_r)| < \epsilon \}$$

in  $\mathcal{B}_{(H_b(U, G)', \sigma(H_b(U, G)'), (H_b(U, G)))}(0)$ . To simplify the notation from now we use  $w^*$  instead of  $\sigma(H_b(U, G)', (H_b(U, G)))$ .

## 2 Composition operators

In this section we study Montel (resp. reflexive) composition operators.

**1 Lemma.** *Let  $E$  and  $G$  be Banach spaces. Let  $a \in G$  and  $a' \in G'$  such that  $\|a\| = 1$ ,  $\|a'\| = 1$  and  $a'(a) = 1$ . Then:*

- (i)  $J_a : (E', |||) \rightarrow H_b(E, G)$  given by  $J_a(x')(x) = x'(x)a$ , for all  $x' \in E'$  and for  $x \in E$  is a continuous linear mapping. Moreover, the transposed mapping  $J'_a : (H_b(E, G)', \tau_\beta) \rightarrow (E'', |||)$  is continuous.
- (ii) Let  $V$  be an open set of  $E$  and let  $\delta_{a'} : V \rightarrow (H_b(V, G)', w^*)$  defined by  $\delta_{a'}(y)(g) = a'(g(y))$ , for all  $y \in V$  and for  $g \in H_b(V, G)$ . Then  $\delta_{a'}$  is a continuous mapping and  $\delta_{a'}$  maps  $V$ -bounded sets into bounded sets for the topology  $\tau_\beta$  on  $H_b(V, G)'$ .

PROOF. (i) It is clear that  $J_a$  is linear. Now, let  $B$  be a bounded set of  $E$  and  $\epsilon > 0$ . So, there is  $\lambda_B > 0$  such that  $\|x\| \leq \lambda_B$  for all  $x \in B$ . If  $x' \in E'$  with  $\|x'\| < \frac{\epsilon}{\lambda_B}$  we have  $\|J_a(x')\|_B \leq \epsilon$  and consequently  $J_a$  is continuous at the origin. For the continuity of  $J'_a$  we consider  $B''_\epsilon(0) \in \mathcal{B}_{E''}(0)$  and the unit ball  $B_{E'}$  of  $E'$ . Then  $J_a(B_{E'})$  is a bounded set in  $H_b(E, G)$  and we have  $|f'(J_a(x'))| < \epsilon$  for all  $x' \in B_{E'}$  and  $f' \in \mathcal{W}_{0, J_a(B_{E'}), \epsilon} \in \mathcal{B}_{(H_b(E, G)', \tau_\beta)}(0)$ . So  $J'_a(f') \in B''_\epsilon(0)$  for all  $f' \in \mathcal{W}_{0, J_a(B_{E'}), \epsilon}$ .

- (ii) Let  $y_0 \in V$  and  $V_{\delta_{a'}(y_0), g_1, \dots, g_r, \epsilon} \in \mathcal{B}_{(H_b(V, G)', w^*)}(\delta_{a'}(y_0))$  with  $g_i \in H_b(V, G)$ . Since each  $g_i$ ,  $1 \leq i \leq r$  is continuous in  $y_0 \in V$ , there exists  $B_{\lambda_i}(y_0) \subset V$  such that for each  $y \in B_{\lambda_i}(y_0)$  we have  $\|g_i(y) - g_i(y_0)\| \leq \epsilon$  for each  $1 \leq i \leq r$ . Let  $\lambda = \min_{1 \leq i \leq r} \{\lambda_i\}$  and  $y \in B_\lambda(y_0)$ . So  $y \in B_{\lambda_i}(y_0)$  and  $|\delta_{a'}(y)(g_i) - \delta_{a'}(y_0)(g_i)| < \epsilon$  for each  $i = 1, \dots, r$ . Consequently  $\delta_{a'}(y) \in V_{\delta_{a'}(y_0), g_1, \dots, g_r, \epsilon}$  and  $\delta_{a'}$  is continuous.

To complete the proof it suffices to show that  $\delta_{a'}$  maps  $V$ -bounded sets of  $V$  into bounded sets of  $(H_b(V, G)', \tau_\beta)$ . Let  $B \subset V$  be a  $V$ -bounded and  $V_{0, g_1, \dots, g_r, \epsilon} \in \mathcal{B}_{(H_b(V, G)', w^*)}(0)$  with  $g_i \in H_b(V, G)$ . So there exist  $\lambda_i > 0$  such that  $\sup_{y \in B} \|g_i(y)\| < \lambda_i$  for  $1 \leq i \leq r$ . Let  $\lambda = \max_{1 \leq i \leq r} \{\lambda_i\}$  and  $g' \in \frac{\epsilon}{2\lambda} \delta_{a'}(B)$ . Then  $|g'(g_i)| < \epsilon, i = 1, \dots, r$  and  $g' \in V_{0, g_1, \dots, g_r, \epsilon}$ . Consequently  $\frac{\epsilon}{2\lambda} \delta_{a'}(B) \subset V_{0, g_1, \dots, g_r, \epsilon}$  and it is  $w^*$ -bounded. Since  $H_b(V, G)$  is a barrelled space we have that  $\delta_{a'}(B)$  is bounded in the space  $(H_b(V, G)', \tau_\beta)$ , thus completing the proof.

QED

In the next theorem we study the Montel composition operator.

**2 Theorem.** *Let  $E, F$  and  $G$  be Banach spaces. Let  $V \subset F$  an open subset,  $\Phi \in H_b(V, E)$  and  $T_\Phi : H_b(E, G) \rightarrow H_b(V, G)$  a composition operator. Consider the following conditions:*

- (a)  $T_\Phi$  is a Montel operator
- (b) The adjoint operator  $T'_\Phi : (H_b(V, G)', \tau_\beta) \rightarrow (H_b(E, G)', \tau_\beta)$  is a Montel operator
- (c)  $\Phi$  maps  $V$ -bounded sets into relatively compact sets in  $E$ .

Then (a)  $\Rightarrow$  (b)  $\Rightarrow$  (c).

PROOF. (a)  $\Rightarrow$  (b) First we show that  $T'_\Phi : (H_b(V, G)', \tau_c) \rightarrow (H_b(E, G)', \tau_\beta)$  is continuous. Let  $\mathcal{X} \subset H_b(E, G)$  be a bounded set and

$$V_{0, \mathcal{X}, \epsilon} = \{f' \in H_b(E, G)' / \|f'\|_{\mathcal{X}} < \epsilon\} \in \mathcal{B}_{(H_b(E, G)', \tau_\beta)}(0).$$

Since  $T_\Phi$  is a Montel operator we have that  $T_\Phi(\mathcal{X})$  is a relatively compact set of  $H_b(V, G)$ . Then the closed absorbing convex hull  $\Gamma(\overline{T_\Phi(\mathcal{X})})$  is compact, since  $H_b(V, G)$  is a Fréchet space.

Now, if we consider

$$W_{0, \Gamma(\overline{T_\Phi(\mathcal{X})}), \epsilon} = \{h' \in H_b(V, G)' / \|h'\|_{\Gamma(\overline{T_\Phi(\mathcal{X})})} < \epsilon\} \in \mathcal{B}_{(H_b(V, G)', \tau_c)}(0),$$

we have that  $T'_\Phi(W_{0, \Gamma(\overline{T_\Phi(\mathcal{X})}), \epsilon}) \subset V_{0, \mathcal{X}, \epsilon}$ , since for each  $h' \in W_{0, \Gamma(\overline{T_\Phi(\mathcal{X})}), \epsilon}$  we get  $|h'(h)| < \epsilon$  for all  $h \in \Gamma(\overline{T_\Phi(\mathcal{X})})$ . So  $|h'(T_\Phi(f))| < \epsilon$  for all  $f \in \mathcal{X}$  and  $T'_\Phi(h') \in V_{0, \mathcal{X}, \epsilon}$ . Therefore  $T'_\Phi : (H_b(V, G)', \tau_c) \rightarrow (H_b(E, G)', \tau_\beta)$  is continuous.

Now, let  $\mathcal{A} \subset H_b(V, G)'$  a  $\tau_\beta$ -bounded set. Since  $H_b(V, G)$  is barrelled we have that  $\mathcal{A}$  is an equicontinuous set and consequently  $\sigma(H_b(V, G)', H_b(V, G))$ -relatively compact. By Banach-Dieudonné theorem we have that  $\mathcal{A}$  is  $\tau_c$ -relatively compact. As  $T'_\Phi$  is  $\tau_c - \tau_\beta$  continuous we have that  $T'_\Phi(\mathcal{A}) \subset H_b(E, G)'$  is  $\tau_\beta$ -relatively compact on  $H_b(E, G)'$ .

(b)  $\Rightarrow$  (c) Let  $a \in G$  with  $\|a\| = 1$ . By Hahn-Banach theorem there exists  $a' \in G'$  with  $\|a'\| = 1$  and  $a'(a) = 1$ . Now, let  $\psi : V \rightarrow (E'', |||)$  defined by  $\psi := J'_a \circ T'_\Phi \circ \delta_{a'}$ . By Lemma 1 we have that  $\psi$  maps  $V$ -bounded sets into relatively compact set of  $E''$ .

As  $\psi(y)(x') = x'(\Phi(y))a'(a) = C(\Phi(y))(x')$ , for all  $y \in V$ , for all  $x' \in E'$  where  $C : E \rightarrow E''$  is the natural inclusion, we have that  $\psi(y) = C(\Phi(y))$  for all  $y \in V$ . So  $C \circ \Phi = \psi$  and  $\Phi$  maps  $V$ -bounded sets of  $V$  into relatively compact sets in  $E''$ . QED

In [3] González-Gutiérrez proved these conditions of Theorem 2 are equivalent when  $G = \mathbb{C}$ . However, in the general case, the following example shows that the assertions of Theorem 1 are not equivalent.

**3 Example.** Let  $\Phi$  be a identity on  $\mathbb{C}$ , which is a trivially Montel mapping. Give an infinite dimensional Banach space  $G$ , we consider the composition operator  $T_\Phi : H(\mathbb{C}, G) \rightarrow H(\mathbb{C}, G)$  given by  $T_\Phi(f) = f \circ \Phi = f$  for all  $f \in H(\mathbb{C}, G)$ . Let  $(y_n)$  be a sequence of norm one vectors in  $G$  such that  $\|y_n - y_m\| \geq \delta > 0$  for all  $n \neq m$ . Define  $f_n : \mathbb{C} \rightarrow G$  by  $f_n(\lambda) = \lambda y_n$ . Then the sequence  $(f_n)$  is bounded in  $H(\mathbb{C}, G)$  but is not relatively compact, since  $(f_n(1)) = (y_n)$  is not relatively compact.

**4 Corollary.** Let  $E, F$  and  $G$  be Banach spaces. Let  $U \subset E$ ,  $V \subset F$  an open set,  $\Phi \in H_b(V, E)$  and let  $T_\Phi : H_b(U, G) \rightarrow H_b(V, G)$  be a composition operator. Then  $\Phi$  maps  $V$  bounded sets into relatively compact sets of  $U$  if  $T_\Phi$  is compact.

PROOF. It suffices to observe that the composition operator  $A : H_b(E, G) \rightarrow H_b(V, G)$  given by  $A(f) = f \circ \Phi$  for all  $f \in H_b(E, G)$  is compact if  $T_\Phi : H_b(U, G) \rightarrow H_b(V, G)$  is compact. QED

The next theorem studies, in the same cases, when the composition operator  $T_\Phi$  is reflexive.

**5 Theorem.** Let  $E, F$  and  $G$  be Banach spaces. Let  $V$  be a balanced open set of  $F$ ,  $\Phi \in H_b(V, E)$  and  $T_\Phi : H_b(E, G) \rightarrow H_b(V, G)$  be a composition operator. Consider the following conditions:

- (a)  $T_\Phi$  is a reflexive operator
- (b) The adjoint operator  $T'_\Phi : H_b(V, G)' \rightarrow H_b(E, G)'$  maps  $\tau_\beta$ -bounded sets into relatively  $\sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)')$ -compact sets
- (c)  $\Phi$  maps  $V$ -bounded sets into relatively weakly compact sets in  $E$ .

Then (a)  $\Rightarrow$  (b)  $\Rightarrow$  (c).

PROOF. (a)  $\Rightarrow$  (b) First we show that  $T'_\Phi : (H_b(V, G)', \tau_\mu) \rightarrow (H_b(E, G)', \tau_\beta)$  is continuous. Let  $\mathcal{X} \subset H_b(E, G)$  be a bounded subset and let

$$V_{0, \mathcal{X}, \epsilon} = \{ f' \in H_b(E, G)' / \|f'\|_{\mathcal{X}} < \epsilon \}$$

be a  $\tau_\beta$ -neighborhood of zero in  $H_b(E, G)'$ . Since  $T_\Phi$  is a reflexive operator it follows that  $T_\Phi(\mathcal{X})$  is a relatively weakly compact set of  $H_b(V, G)$ . As  $H_b(V, G)$  is a Fréchet space we have that the closed convex absolutely hull of  $T_\Phi(\mathcal{X})$ ,  $\Gamma(\overline{T_\Phi(\mathcal{X})})$ , is a weakly compact set of  $H_b(V, G)$ .

Now, we consider  $W_{0, \Gamma(\overline{T_\Phi(\mathcal{X})}), \epsilon} = \{ h' \in H_b(V, G)' / \|h'\|_{\Gamma(\overline{T_\Phi(\mathcal{X})})} < \epsilon \} \in \mathcal{B}_{(H_b(V, G)', \tau_\mu)}(0)$ . Then it is clear that  $T'_\Phi(W_{0, \Gamma(\overline{T_\Phi(\mathcal{X})}), \epsilon}) \subset V_{0, \mathcal{X}, \epsilon}$ . We claim that

$$T'_\Phi : (H_b(V, G)', w^*) \rightarrow (H_b(E, G)', \sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)'))$$

is a continuous mapping. Indeed let

$$V_{0, f''_1, \dots, f''_k, \epsilon} \in \mathcal{B}_{(H_b(E, G)', \sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)'))}(0)$$

with  $f''_i \in (H_b(E, G)', \tau_\beta)'$  for each  $i = 1, 2, \dots, k$ . Then we have that  $T''_\Phi(f''_i) \in (H_b(V, G)', \tau_\mu)'$ . Thus there exist  $f_i \in H_b(V, G)$  such that  $T''_\Phi(f''_i)(f') = f'(f_i)$  for all  $f' \in H_b(V, G)'$  and  $i = 1, 2, \dots, k$ . If we consider

$$W_{0, f_1, \dots, f_k, \epsilon} \in \mathcal{B}_{(H_b(V, G)', \sigma(H_b(V, G)', H_b(V, G)))}(0),$$

it is easy to see that  $T'_\Phi(W_{0, f_1, \dots, f_k, \epsilon}) \subset V_{0, f''_1, \dots, f''_k, \epsilon}$  and consequently  $T'_\Phi$  is  $\sigma(H_b(V, G)', H_b(V, G)) - \sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)')$  continuous.

Now, let  $\mathcal{X} \subset H_b(V, G)'$  be a  $\tau_\beta$ -bounded set. As  $H_b(V, G)$  is barrelled we have that  $\mathcal{X}$  is a  $w^*$ -relatively compact.

So  $T'_\Phi(\mathcal{X})$  is  $\sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)')$ -relatively compact, and the implication (a)  $\Rightarrow$  (b) follows.

(b)  $\Rightarrow$  (c) Let  $a \in G$  with  $\|a\| = 1$ . Then there is  $a' \in G'$  such that  $\|a'\| = 1$  and  $a'(a) = 1$ .

First we show that

$$J'_a : (H_b(E, G)', \sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)')) \rightarrow (E'', \sigma(E'', E'))$$

is a continuous mapping at origin, where  $J'_a$  is the adjoint operator of  $J_a$  defined in the Lemma 1.

Let  $V_{0, x'_1, \dots, x'_r, \epsilon} \in \mathcal{B}_{(E'', \sigma(E'', E'))}(0)$ . For each  $i = 1, 2, \dots, r$ , we consider  $x'_i \otimes a \in H_b(E, G)$  defined by  $x'_i \otimes a(x) = x'_i(x)a$  for all  $x \in E$  and  $f''_i : (H_b(E, G)', \tau_\beta) \rightarrow \mathbb{C}$  given by  $f''_i(f') = f'(x'_i \otimes a)$  for all  $f' \in H_b(E, G)'$ . Now, it holds that  $W_{0, f''_1, \dots, f''_r, \epsilon} \in \mathcal{B}_{(H_b(E, G)', \sigma(H_b(E, G)', (H_b(E, G)', \tau_\beta)'))}(0)$  and we have  $J'_a(W_{0, f''_1, \dots, f''_r, \epsilon}) \in V_{0, x'_1, \dots, x'_r, \epsilon}$ . So by Lemma 1 (ii) the mapping  $\psi : V \rightarrow$

$(E'', \sigma(E'', E'))$  given by  $\psi = J'_a \circ T'_\Phi \circ \delta_{a'}$  maps  $V$ -bounded set into  $\sigma(E'', E')$ -relatively compact set in  $E''$ .

Let  $(y_n)_{n \in \mathbb{N}} \subset V$  be a bounded sequence. Since  $\psi$  is  $\sigma(E'', E')$ -relatively compact and  $\psi(y)(x') = C(\Phi(y))(x')$  for all  $y \in V$  and  $x' \in E'$  where  $C$  is the natural inclusion from  $E$  into  $E''$ , there exist a subsequence  $(y_{n_k})_k$  of  $(y_n)$  and  $x'' \in E''$  such that  $x'(\Phi(y_{n_k})) \rightarrow x''(x')$  for all  $x' \in E'$ . Consequently,  $\Phi$  maps bounded sets of  $V$  into  $\sigma(E, E')$ -relatively compact sets in  $E$ .  $\square$

**6 Remark.** Slight modifications of example 1 give an example that shows in general the assertions of Theorem 2 are not equivalent. Indeed, let  $\Phi$  be a reflexive mapping and let  $G$  be a non-reflexive Banach space, and choose a sequence  $(y_n)$  of norm one vectors in  $G$  without any weakly convergent subsequence. Define  $(f_n)$  as the example 1. Then  $(f_n)$  is not relatively weakly compact.

In [4] M. González and J. Gutiérrez showed that the conditions of Theorem 2 are equivalent if  $G = \mathbb{C}$  and  $E$  has Dunford-Pettis property.

## References

- [1] S. DINEEN: Complex Analysis on Infinite Dimensional Spaces, Springer-Verlag, London Berlin Heidelberg 1999.
- [2] P. GALINDO, M. L. LOURENÇO, L. A. MORAES: *Compact and weakly compact homomorphisms on Fréchet algebras of holomorphic functions*, Math. Nachr., **236**, (2002), 109–118.
- [3] M. GONZÁLEZ, J. GUTIÉRREZ: *Schauder type theorems for differentiable and holomorphic mappings*, Monatsh. Math., **122**, (1996), 325–343.
- [4] M. GONZÁLEZ, J. GUTIÉRREZ: *Gantmacher type theorems for holomorphic mappings*, Math. Nachr., **186**, (1997), 131–145.
- [5] J. HORVATH: Topological Vector Spaces and Distributions, Vol. 1. Addison-Wesley, Massachusetts 1966.
- [6] H. JARCHOW: Locally Convex Spaces, Mathematische Leitfäden, B. G. Teubner, Stuttgart 1981.
- [7] J. MUJICA: Complex Analysis in Banach Spaces, Math. Stud., **120**, North-Holland 1986.